What Henry Cavendish Didn't Tell Us About His Gravity Experiment

July 14, 2024

By: Arthur L. Roberts

BE, Yale 1962

Sigma XI 1962

MBA, Harvard 1967

In 1798 Henry Cavendish presented a paper to the Royal Society London in which he described his Experiment to Determine the Density of the Earth, a number that was needed to accurately calculate planetary orbits. In modern physics, the same experiment can be used to calculate Big G, the Universal Gravitational Constant.That number has many uses today in astronomy and calculating trajectories for satellites and for landing objects on distant planets and moons. The generally accepted value for G is 6.7743×10^{-11} $m^3kg^{-1}s^{-2}$, a devilishly small number. While Cavendish solved for the density of the earth, his data can just as easily be used to solve for G. Using his data the result is within about one percent of today's generally accepted value of G. That achievement is truly remarkable. Cavendish obtained very close correlation of exceedingly small measurements in what today would be considered crude circumstances. His 54 page paper, containing diagrams of the apparatus, descriptions of his procedures and analysis of possible sources of error set a standard for clarity and completeness for the nascent scientific community of the late18th century.

CAVINDISH'S APPARATUS

Cavendish employed a torsion balance in his apparatus. The following diagrams show the apparatus as seen from above, front view and detail of the beam and small balls.

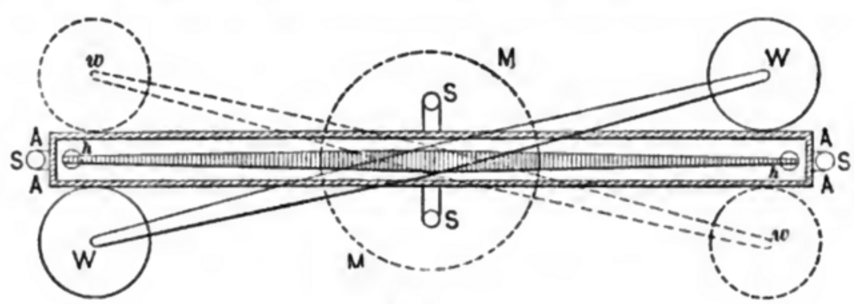
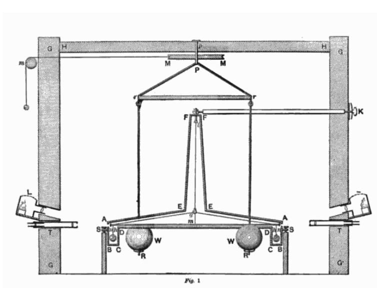
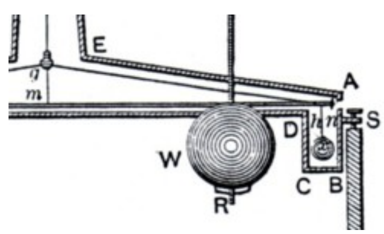


Fig. 2



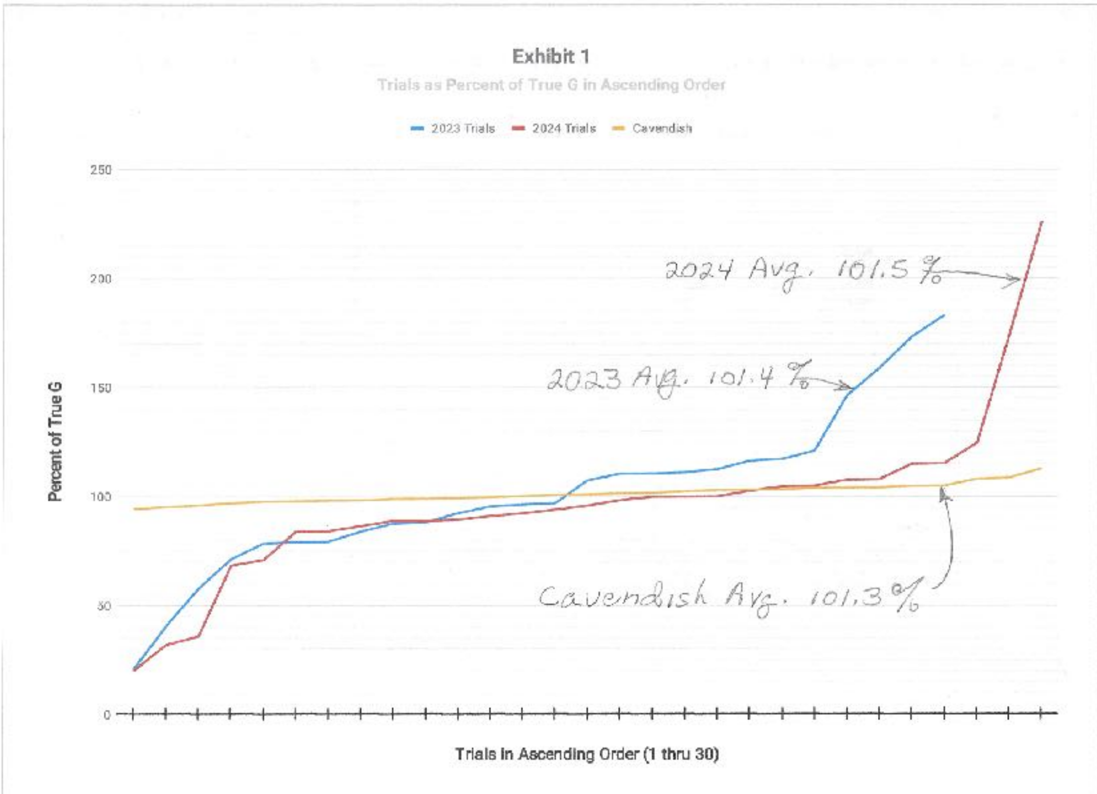


It consisted of a thin light weight beam suspended horizontally from a single wire with small lead weights (h,h) hanging from either end. Two much larger weights (W,W) could be swung near the the small balls to provide a gravitational force on the small balls which would cause the beam to slightly rotate. The large weights could be rotated to the other side of the torsion beam, causing the beam to rotate in the opposite direction. By measuring the angular deflection of the beam he was able to calculate the resistance of the wire supporting the beam and thus measure the forces. The beam and small balls were encased in a cabinet to prevent air currents from affecting the results.

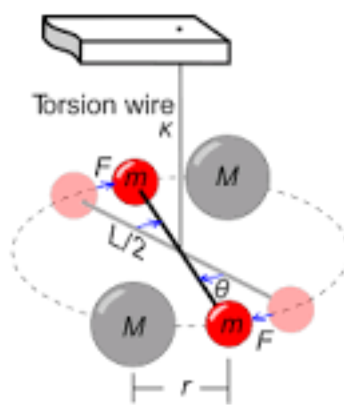
MY ATTEMPTS TO REPLICATE THE CAVENDISH EXPERIMENT

Out of curiosity several years ago I began my own effort to replicate the Cavendish experiment. My initial objective was to simply see whether I could detect gravity. Those early trials were disappointing as I came to appreciate how sensitive the experiment is to the configuration of the equipment, the environment and procedure. Eventually I achieved enough success to set a goal of reaching within 10% of G. There were many difficulties, including locating large enough spherical weights, finding an appropriate wire from which to suspend the torsion beam, building an airtight box to house the torsion balance and working around daily temperature changes. Cavendish's paper makes no mention of his encountering any difficulties; my experience suggests that it is highly likely that he did.

Exhibit 1 shows Cavendish's and my results in graphical format. I have chosen to arrange these results in ascending order to illustrate their dispersion from the ideal 100% line which represents the generally accepted value for G. Cavendish reported a total of 29 trials for an average of 101.3% of the generally accepted value of G. The average of my 26 results in 2023 is 101.4%. The average of my 30 trials in 2024 is 101.5%. Cavendish's results are more tightly clustered with a standard deviation of .042%. While my results for both years are well within my stated objective of + or -10%, my hat goes off to Cavendish because he achieved a much smaller dispersion. This difference is indicative of the difficulties I encountered. However it is also likely Cavendish encountered similar problems until he sufficiently refined his apparatus and procedures. My experience leads me to question Cavendish's story, not to suggest that he distorted the facts, rather I believe he chose not to mention the improvements that were necessary before achieving his unquestioned success. I will return to this subject after describing my apparatus and operating procedure.



THEORY A torsion balance is an especially effective mechanism to measure very small forces. As the diagram at the right shows, when large masses M and M are placed near the small masses m and m, the gravitational forces F and F cause the beam hanging from the torsion wire to rotate toward the large masses by θ angle until the torsion wire presents a resisting torque equal to the torque resulting from the forces F and F times the lever arm L/2. The resistance of the wire can be expresses as θ times k, the torsion coefficient of the wire. When the motion has stabilized at θ, the following equations apply:



$$\theta k = 2\,\frac{(FL)}{2} \tag{1}$$

F may be expressed as:

$$F = \frac{MmG}{(r^2)} \tag{2}$$

Substituting for F equation (1) may be restated as:

$$\theta k = \frac{MmLG}{(r^2)} \tag{3}$$

The torsion coefficient of the wire can be determined by measuring the resonant oscillation period T of the torsion balance.

$$T = 2\pi\sqrt{I/k} \tag{4}$$

Assuming the mass of the beam to be negligible, the moment of inertia of the torsion balance is:

$$I = m(L/2)^2 + m(L/2)^2 = mL^2/2 \tag{5}$$

Substituting for I in equation (4), solving equation (4) for k, substituting for k in equation (3) and solving for G:

$$G = \frac{2\pi^2 L r^2 \theta}{MT^2} \tag{6}$$

Since the mass of the the torsion beam is not negligible there are two adjustments that must be made to this basic formula. The first is to account for the moment of inertia of the beam which is $I_{beam} = (m_{beam}L^2)/12$ where m is the mass of the beam and L is its length. This has the effect of increasing G by $(1+I_{beam}/I_{ball})$. The second correction is to account for the gravitational attraction of the torsion beam toward the large weights. This has the effect of decreasing G. The effective mass of the torsion beam on a point at the center of the large mass is $(m_{beam}r)/\sqrt{L^2+4r^2}$, where r is the distance between the beam and the external lead weight. Since all these numbers are known the effect for my setup using .913 Kg spheres is 1.061/1.033 = 1.0276 times

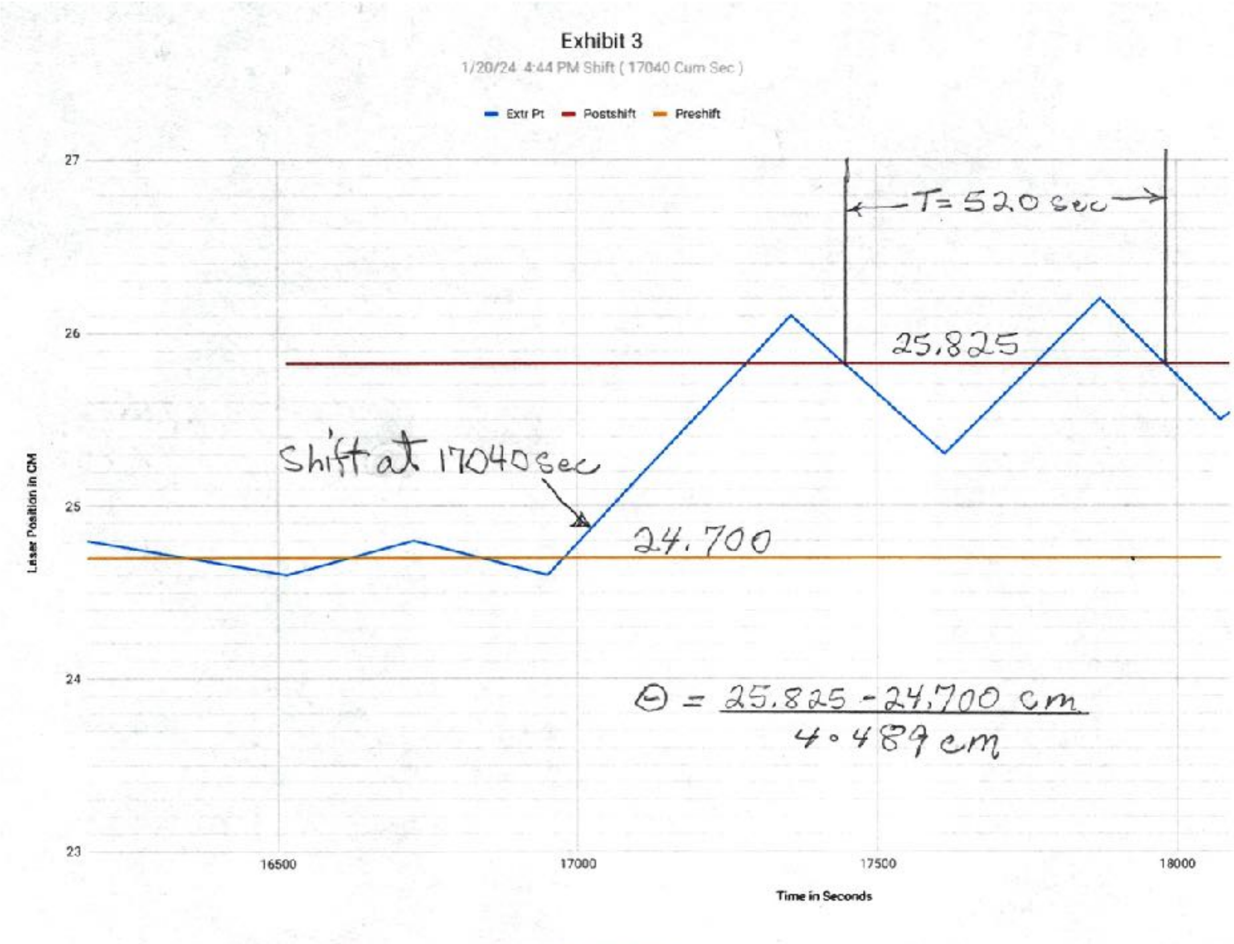
G calculated from the above formula. For the set of lighter spheres weighing .528 Kg which were used in some trials the effect is 1.106/1.056 = 1.047 times G.

SETUP I employed a torsion balance using a variety of steel wires or thin copper strips from which the torsion beam is suspended and several sizes of small lead spheres at the ends of the beam. Using one combination as an example the beam is suspended from a copper ribbon approximately one meter long, .1 mm thick and 10 mm wide. The beam is a pvc pipe approximately 1.9 meters long and weighing .277kg. At either end of the beam are hung spherical lead balls each weighing .913 kg. This entire arrangement is enclosed in a narrow wooden box to protect the apparatus from air currents. Suspended from either end of a separate overhead beam external to the box are two 36.74 kg (81 pound) spherical lead attractor weights which can be brought next to the box so as to impart a gravitational force on the small balls at a right angle to the torsion beam in the box. The beam from which the large weights are suspended can be rotated about a center aligned with the center of rotation of the torsion beam. This enables the gravitational forces to be employed to pull the balls alternately clockwise and counterclockwise. A laser beam pointed at a mirror mounted at the base of the copper ribbon provides a means for measuring θ, the amount of angular deflection caused by drawing the large weights close to one side of the box and then shifting the large weights to the other side of the box. The entire apparatus is contained in a small room with the large weights able to be shifted from the left position to the right position, and vice versa, by means of ropes to avoid human intrusion in the room which would introduce air currents and disturb the gravitational field. The laser beam reflects off the mirror attached to the bottom end of the copper strip and registers on a target 4.89 meters from the center of rotation of the torsion beam. Cavendish employed a scale at the end of his beam to measure θ. Exhibit 2 is a tabular comparison of key elements of Cavendish's and my apparatuses.

Exhibit 2

Definition	Symbol	Unit	Cavendish	Roberts
Universal Gravitational Constant	G	$\frac{m^3}{Kgsec^2}$		
Mass of Small Balls	m	Kg	0.73	.913 .528
Distance Between Small Balls	L	m	1.86	1.83
Mass of Large Attractor Balls	M	Kg	158.04	36.74
Mass of Torsion Beam	m_{beam}	Kg	0.155	0.277
Closest Distance Between centers of Small and Large Balls	r	m	0.225	0.1795
Period of Oscillation	T	Sec		
Angular Movement of Torsion Beam	θ	Radians		

PROCEDURE The laser beam reflects off the mirror and registers on grid paper which can be read to the nearest 1 mm. A GoPro video camera is placed about 50 cm in front of the target grid. The GoPro is set up to record an image every two seconds and when played back it is easy to discern the maximum left and right travels of the laser for each cycle. Cavendish called the max left and right travels extreme points and the midpoint of each cycle the point of rest. Since the beam is seldom entirely stationary the first step in performing a trial is to record video for at least an hour to establish the starting midpoint position. The next step of the trial is to use the ropes to remotely move the large weights from the left side to the right side, or vice versa, taking care to position the weights as close to the walls of the box without impacting the box (which would cause a percussive air current inside the box). Another hour of continued recording reveals that the oscillations have shifted to the right or left, depending on which direction the weights were moved. The shift in the point of rest divided by the distance from the mirror to the target (4.89 meters) is an angle that is divided by 4 to obtain θ. (The measured angle must be divided by 2 to account for the doubling caused by rotation of the mirror and again by 2 to reflect that my trials pull the beam to one side and then the other, which is effectively two gravitational effects.) The length of cycle (T seconds) can be determined by reading a digital clock which is stationed within the view of the GoPro camera. Exhibit 3 is a graphical depiction of the data for one trial.



ANALYSIS Because the force of attraction between the large attractor weights and the small balls is very small, it is desirable to maximize that force by increasing the mass of the attractor weights and small balls and by minimizing the distance between them (r). The latter can be accomplished by reducing the width of the box as much as possible while still allowing free oscillation of the beam and balls. I tried a variety of balls on the torsion beam ranging from 1.78 kg (4 lb) to .53 (1.2 lb) but found that .917 kg and .53 kg worked best. I found the increased moment of inertia of heavier balls increased irregularity of θ and T. No such penalty is paid by increasing the big exterior attractor weights. I began with 12 pound (5.44 kg.) shot put spheres borrowed from a local high school, advanced to 35 pound spheres and eventually found a source for two 36.74 kg. (81 pound) lead spheres. Performance improved significantly with each increase of the big weights. All of my trials reported here employed the 36.74 kg. weights. In contrast all of the trials which Cavendish described in his paper employed 158 kg (347 pound) lead spheres. Handling those heavy weights must have been a challenge for his workers but the results were worth it. When he shifted the weights from one side to the other the point of rest of the small balls shifted by about 7 mm or 14 mm, depending on the diameter of the wire suspending his beam. In my setup the typical shift was 1 to 2 mm. Larger movement reduces the effect of distortions and measurement anomalies.

Relative to Cavendish, my apparatus has about a 12 to 1 advantage in accuracy of measuring θ. Cavendish could read the scale at the end of his torsion arm to 1/20th of an inch. The scale was located .975 meters from the center of rotation of the torsion arm resulting in resolution of 1.30×10^{-3} radians. My laser target is 4.89 meters from the center of rotation and can be read to .001 meters. Because of the doubling effect of the mirror this is effectively .0005 meters for a resolution of 1.02×10^{-4} radians. In a typical trial when Cavendish shifted the weights from one side to the other the point of rest of the small balls shifted by about 3/10 of an inch or 7.8×10^{-3} radians. In my setup the typical shift was 20 mm. or 2.0×10^{-3}. While I could read small movements more accurately, Cavendish had the advantage of reading larger movements. This reduces the effect of distortions and measurement anomalies. Cavendish's use of enormous weights is likely the single biggest reason for his better consistency.

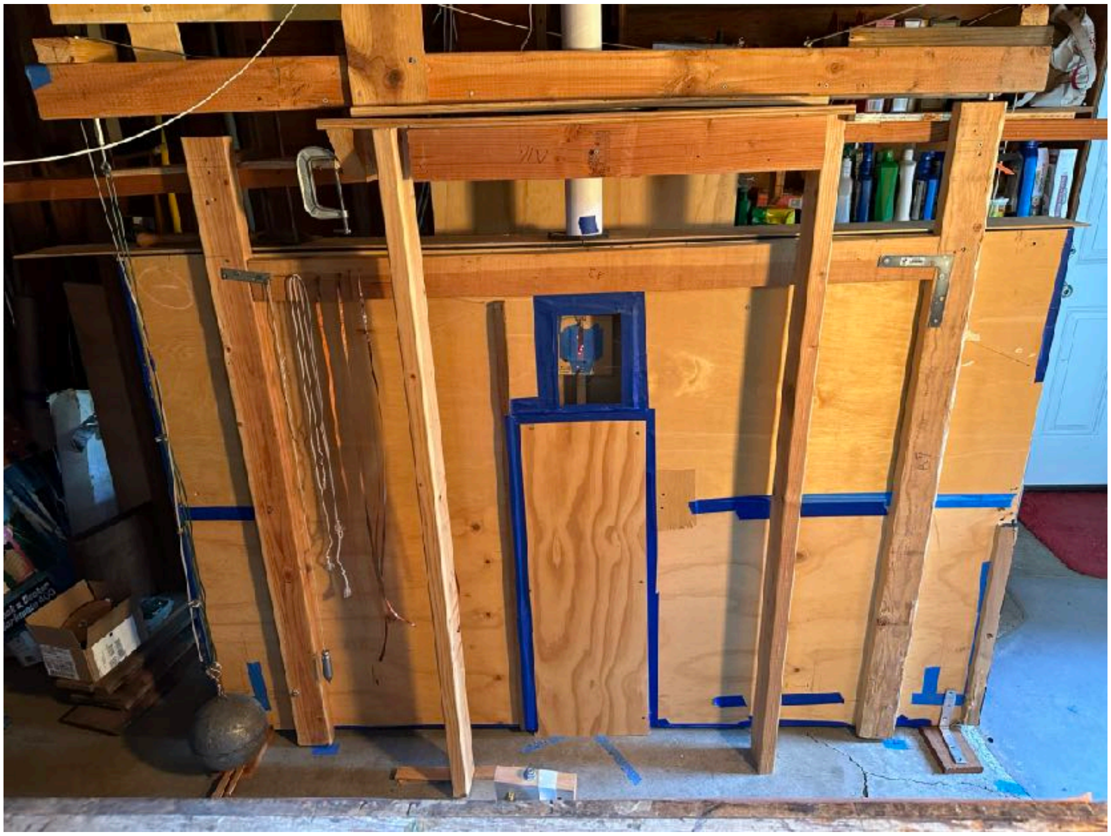
Another source of error is ambient temperature change. This affects the stiffness of the steel wire or copper strip from which the torsion beam hangs. Like Cavendish's, my apparatus is housed in a utility building with minimal insulation and no means of heating or cooling. I found it best to conduct trials during overcast or rainy days since there is less solar heating of the exterior of the building. I also found it best to limit my trials to late afternoon through early evening when the daytime warming had peaked and cool off had not begun in earnest. My best results came on rainy or overcast days when the temperature inside the box varied by no more than 1 degree Centigrade.

A final source of error is the box enclosure. I found it necessary to tape all the joints and seal all access ports with insulating foam to prevent air currents disturbing the torsion beam. I also installed bumper stops close to the box (but not touching) to guard against the heavy weights bumping the wall of the box and setting up a concussive air wave which would disturb the torsion

beam. Cavendish employed bumper stops as well.

Examination of my lowest and highest results as seen in the downward and upward tails in Exhibit 1 reveals no consistent pattern. A deviant result typically occurred within a batch of very acceptable trials on the same day. I attribute these outliers to the previously mentioned anomalies that result from small θ. As I can find no pattern to provide a rational for eliminating the more extreme results, I have left them in. It is worth noting however, that if my 2023 and 2024 results are combined and the 10 lowest and 10 highest data points are eliminated the average for the remaining data set is 97.7% with a standard deviation of 10.8%.

CONCLUSION Cavendish clearly got it right and gave the young scientific world a model for how to conduct scientific enquiry with thoroughness and clarity. His "Experiments to

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Determine the Density of the Earth" is known throughout the world as "The Cavendish Experiment", a testament to his immense contribution to Physics and the development of the scientific method.

Arthur L. Roberts
arthur@robdog.com

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